The Final Grant Report for the DC proposal Math 193: Pedagogy and Teaching Disciplinary Communication course for the Mathematics Education Track Majors to Support the Hybrid Teaching Model of Precalculus (Math 3)

Objective of the proposal:

The primary objective of the proposal was to develop an upper division "Pedagogy and Teaching DC course" (Math 193 AB) especially geared for the Education track Mathematics majors, which they can take instead of Math 194 or 195.

Please see the attached syllabus for Math 193AB. The syllabus describes the weekly goal in detail with the leading question for the session. It also specifically focuses the weekly reading for the course that augments the discussion. Textbooks for the course were selected and other specific articles pertinent to the subject were gathered from many sources, including various journals in Mathematics Education as well as the LA program alliance at the CU Boulder campus.

The six-unit course taught over two quarters will have a theoretical and a practical component. We will discuss effective teaching methods in a weekly seminar-style class. Students will be able to apply that knowledge in the Math 3 discussion sections in collaboration with the graduate student TAs.

The heart of precalculus is the concept of a function – a rule assigning a mathematical object to each member of some collection of mathematical objects. In Precalculus, the objects in question are real numbers. One example of such a function studied in Math 3 are Linear Functions. In week 2 and week 6 in the pedagogy course, specific articles will be discussed in conjunction with the learning theory to show the direct connection between the theory and practice of these specific topics. Moreover, as the students develop their mini lesson plans (weeks 4-10) with problem-based worksheets, we will connect the learning theory of education with practical implementation into the Precalculus curriculum. In Math 3 students study different families of functions such as Linear, Quadratic, Polynomial, Exponential, Logarithmic and Trigonometric functions. In Math 193AB students will choose one such family of functions and develop and implement a mini lesson plan with a complete worksheet by incorporating best practices of Cooperative Learning teaching methods. They will learn to facilitate group work by asking questions, from how to process facts such as definitions and theorems, to basic mathematical skill building such as solving and simplifying to generalizing the underlying main concepts as a theme, to examining how it can be applied to real life problems.

The secondary objective of the proposal is the development of a Hybrid Precalculus lecture course supported in part, by undergraduate students enrolled in the pedagogy course, as well as graduate student TAs. Hybrid instruction offers an effective pedagogically sound solution to the challenge of teaching Precalculus to an extraordinarily large and diverse cohort of students. We will break down the curriculum into concept modules. For example, after we introduce the techniques of graphing by using transformation, we will introduce the students to different families of functions (e.g. logarithmic, exponential, trigonometric) studied in Precalculus, and show them how the concepts related to transformation of graphs are common to all of these families. The same idea will be pertinent to solving equations, applying them to real world problems, etc. By following this approach, rather than closely following the structure of a standard textbook, we will enhance students' learning by emphasizing mastery of concepts over memorization of property after property.

The curriculum development for the secondary objective as described below:

- A new textbook for Math 3 was chosen with a team of colleagues from the department. I have directly worked with many of the publishers and short-listed our choices. The book was chosen to satisfy the pre-requisites of the Calculus course, and comes with state of the art online resources.
- The Math 3 lecture material was developed to teach the course in active learning format. I piloted this non-traditional method of lecture delivery during Winter and Spring quarters of 2016. The content of the course was delivered by combining direct teaching methods with having students work on carefully chosen problems with another peer (Think-Pair-Share Cooperative Learning method). The LSS tutors were available and walked around the lecture hall to help students with their questions. All together there were 31 sessions completed during Spring quarter of 2016.
- I have worked with Sheryl Martin-Schultz at FITC to establish a secure connection using either Reef Polling software or I-clickerx during Math 3 lecture. Students had a choice as to whether they wanted to use their smart phone with Reef or the I-clicker that they have already purchased for another class. I have been successful in establishing this technology both in classroom unit 2 (450+ students) and in Thimann 3 (140 students). The students received 5% of their overall grade both quarters for actively participating during lecture.
- A course website for Math 3 was created using Canvas. The website for this course is **https://login.uconline.edu/** under Math 3 Spring16. Given that this is a fast paced course, the curriculum was broken down by each week of the quarter according to the lecture schedule for students to easily access the curriculum. The

entries for each week consisted of short lecture videos, worksheets, lecture podcasst, the uploaded class notes for the week, etc. **Please see the attached screen shot of the Math 3 LMS.**

- During the Math 3 lecture, students answered quiz-like questions (with multiple parts) as part of their lecture participation using Reef/I-clicker. Students will also take an on line quiz at the end of each short lecture video to show basic level comprehension of the topic introduced. However, it is also important for students to take quizzes with the traditional pen and paper process because only then can we encourage them to show how they reached their solution and not just have them answer multiple choice questions. Showing work on their quiz also enabled TAs to give evaluations on any conceptual mistakes made by the students early in the course. Students took their quizzes in discussion sections and they were graded regularly. Students got feedback on their work on a regular basis.

The proposed hybrid course is tailored to accommodate diverse learning styles and to allow students to learn at their optimal pace, determined by their own learning needs. We intend to present most of the definitions, theorems, and basic examples in on-line 10-15 minute modules in the form of lecture videos, allowing more time in class to develop the material in greater depth via discussion, examples, and questions students answer with a clicker system after they view the modules.

I have created 35 short lecture videos on the conceptual groundwork of the Math 3 lectures at FITC with the help of Aaron Zachmeier. **Please see the attached letter from Aaron which explains the technical components of the project that we needed to figure out**. It also explains how we can further improve students' participation as they watch the videos, by designing interactive quizzes based on the content of the video to check for students' understanding. The length of each video is anywhere from 10 minutes to 25 minutes depending on the topic. **Please see the attached list for more details.**

The timeline of the course development:

Summer 2015: The new Precalculus book was selected. The online homework component using Mathlab was determined. Research for the reading material for Math 193AB began as I worked closely with Judit Moschkovich. I spent a lot of time reading the two textbooks chosen for Math 193AB and developed ideas on how to connect the best practices of educational theory with the teaching of Precalculus. The syllabus for Math 193AB was created.

Fall 2015:

- The lecture notes and the worksheets for the Math 3 course were developed using the new textbook that would promote active learning teaching in the large lecture. The delivery of the curriculum had to be interactive, balancing between direct instruction and active student participation with carefully selected questions during lecture.
- I also developed all the worksheets and their solutions for the collaborative learning sections during this quarter.
- The initial planning on developing the short lecture videos with the help of Aaron Zachmeier also took place. We figured out what video recording technology would best serve the purpose of creating the Tablet-based short lecture videos.

Spring 2016: Thirty-five short pre-lecture videos were created using Camtasia 2. They were created in FITC with Aaron Zachmeier. In each module, we started with objectives of each of the module. After we introduced the topic of the video with a proper mathematical foundation using definitions and theorems, appropriate examples were shown to demonstrate and show the theory in action. When the lecture video was on transformation of function, the example was graphical; for solving of logarithmic and exponential equations, we chose the examples to be more computational; and to show how logarithmic and exponential functions can be used in real life, we chose the representative example to be applied. Please see the attached list for more details on the breakdown of the topics for each video.

We also created all the exam reviews and their corresponding solutions during this quarter.

How the delivery of the Math 193AB can be improved: As I developed this course last year and the department attempted to offer it last spring, we encountered some resistance due to it's low enrollment. The major problem was that a six-unit two-quarter sequence made the course less attractive to the graduating seniors–too few credits spread over too many quarters. As a result only 6 students signed up instead of 8 and the course had to be cancelled. If this course was to be offered in the future, I feel we need to make the following revisions:

1) Make it a 5 units course offered in one quarter (either winter or spring).

2) Advertise the course in the fall quarter and let all the relevant advisors know about it ahead of time instead of trying to solicit student enrollment via emails last minute.

How the project can be scaled: The scope of this project is quite broad. We can use the 5 unit pedagogy course to replace Math 188, which currently is hard to offer given the restrictions required by the TA union contract. We can use it as a course for the CalTeach students where they will get to work with University students and gain knowledge about best practices of teaching and learning. It can be also scaled down to be a 2 unit TA training course for the Math department. This course can be also used to train the undergraduate learning assistants for Learning Support Services. Math 193 can be offered as a DC elective course for the Education Track Math majors if the department perceives this as not appropriate as a substitute for the senior seminar class. In that case we can broaden the target population of Math 193 by including other STEM majors such as Physics, AMS etc. Lastly, I am open to working with interested faculty to make it better suited to be a senior seminar course by changing the level of Mathematics to a higher level of rigor..

Impact of the project: **The** Mathematics department serves a huge number of students taking College Algebra, Precalculus and Calculus courses. However, currently the department does not have any course where we are helping the graduate students learn the best practices of teaching. This course will give a major boost to the beginning of the quarter TA training effort. It will provide the TAs solid training on teaching and mentor them throughout the quarter as a way to improve their teaching skill sets. Since many graduate students in Math pursue teaching careers, it will be an important skill to hone for them. The message and culture of valuing teaching by the department will be communicated with the offering of such a course. In the long run, we can also use it to train TAs in STEM, as they do in the Learning Assistant program at CU Boulder.

Syllabus for Math 193A and 193B
Senior Seminar for the Education Track Math Majors
Nandini Bhattacharya

Course Description and Overview

Math 193AB, a seminar course, will provide an introduction to effective teaching methods specifically geared towards the Precalculus (Math 3) curriculum. The course will address the theory and practical aspects of teaching the gateway course Math 3. Significant time will be dedicated to discussions on learning effective **communication** in the teaching of Precalculus (Math 3). The pedagogical discussion will be based on learning group facilitation techniques, specifically focused towards cooperative learning. Specific topics covered will be classroom strategies, designing assessment activities, constructing cooperative learning activities, etc., using examples directly drawn from the Precalculus curriculum. At the heart of the Math 3 class is the study of Functions: Linear, Quadratic, Polynomial, Rational, Logarithmic, Exponential and Trigonometric Functions. Worksheets to be solved in the Precalculus (Math 3) discussion sections will be developed and used, incorporating the critical thinking posed by the pedagogy curriculum in Math 193AB.

Required Books and Reading Material:

- *5 Practices for Orchestrating Productive Mathematics Discussion* by Margaret Schwan, Smith Mary, Kay Stein.
- *Cooperative Learning in Undergraduate Mathematics Issues that Matter & Strategies that Work* by Elizabeth C. Rogers, Barbara E. Reynolds, Neil A. Davidson, Anthony D. Thomas.
- *Precalculus, Concepts through Functions. A Unit Circle Approach to Trigonometry* by Sullivan and Sullivan.
- Selected articles will be available on Canvas for additional reading

Assignments and Grading Policy

1. Weekly Journals: Writing short responses (1-2 pages) to questions using the reading materials and lectures, answering the focus questions pertinent to the reading for that week and participating in the class discussions. Weekly journals are due at the end of each class. (25 points)

2. Presentation: Develop and present a mini lesson using the main concept from the Precalculus course, reflecting the reading and strategies learned in the pedagogy course. Students will develop a Math 3 worksheet as an example to demonstrate the theory. The format of the presentation of the concept will be based on the cooperative learning styles of teaching. (25 points).

3. Final Portfolio: Submit a portfolio of writing (18-20 pages) consisting of the key reflections and learning outcomes from the weekly journal in the course, lesson plans, and worksheets created for the Math 3 discussion sections. This final assignment will

demonstrate a cohesive and thematic development of a math topic based in educational theory, reflective of the best teaching practices, with a clear thesis. This assignment will be due at the end of the last class. (50 points)

Course Schedule:

Week 1:

Orientation and Overview: Introduction of the themes and purpose of the course. Going over the Math 3 syllabus and all the online features of the Math 3 course. Explaining how the Math 3 course will be used as a practicum for applying the pedagogical theory discussed and learned in the Math 193AB course. Discuss the reading materials and assignments for the class.

Week 2:

Discussion Techniques-Univocal/Dialogue Discussion. Introduction of the the Five Practices Questioning Strategies and Question Types. Developing an in-class activity with the concept of function, reflective of the 5 practices: Anticipating, Monitoring, Selecting, Sequencing and Connecting.

Required readings:

- *5 Practices for Orchestrating Productive Mathematics Discussion* by Margaret Schwan, Smith Mary, Kay Stein, pages 1-12.
- Article on "Functions, Graphs, and Graphing: Tasks, Learning and Teaching" by Gaea Leinhardt, Orit Zaslavsky and Mary Kay Stein.

Week 3:

Discussion on how to set clear learning goals for a lesson and how to choose appropriate tasks to reach that goal. How to provide students with the opportunity to engage in high-level thinking; how to use Cooperative Learning methods to enhance individual students' learning; how interactions with each other can motivate students both internally and externally.

Required readings:

- *5 Practices for Orchestrating Productive Mathematics Discussion* by Margaret Schwan, Smith Mary, Kay Stein, pages 13-19.
- *Cooperative Learning in Undergraduate Mathematics Issues that Matter & Strategies that Work* by Elizabeth C. Rogers, Barbara E. Reynolds, Neil A. Davidson, Anthony D. Thomas, pages 1-13
- *Article on A Designer Speaks: Designing a Multiple Representation Learning Experience in Secondary Algebra by Malcolm Swan*.

Week 4:

Starting to develop a mini-lesson plan and a worksheet for the Math 3 discussion sections. Investigating the Five Practices in Action. Carefully examining practical implementation issues within the individual classroom. Classroom Strategies for Cooperative Learning. Laying out the basic structure of a lesson plan.

Required readings:

- *5 Practices for Orchestrating Productive Mathematics Discussion* by Margaret Schwan, Smith Mary, Kay Stein, pages 21-29.
- *Cooperative Learning in Undergraduate Mathematics Issues that Matter & Strategies that Work* by Elizabeth C. Rogers, Barbara E. Reynolds, Neil A. Davidson, Anthony D. Thomas, pages 14-23.
- *Article on Collaborative Learning in Mathematics*: a personal perspective by John A. Beachy.

Week 5:

Designing assessment activities to encourage productive collaboration. Anticipating students' responses and monitoring their work. How to assess students' work during sections in order to better help them collaborate with each other to gain conceptual understanding.

Required readings:

- *5 Practices for Orchestrating Productive Mathematics Discussion* by Margaret Schwan, Smith Mary, Kay Stein, pages 31-42.
- *Cooperative Learning in Undergraduate Mathematics Issues that Matter & Strategies that Work* by Elizabeth C. Rogers, Barbara E. Reynolds, Neil A. Davidson, Anthony D. Thomas, pages 24-55.

Week 6:

Learning theory and constructing Cooperative Learning Activities. Design and development of the lesson plan. Students will be guided through construction of a sample worksheet where we will develop questions emphasizing various layers of difficulty and thinking process. They will see samples of questions using facts (definition, theorem, data), skills (proficiency with mechanics such as factoring, solving, simplifying), concepts (underlying connection among multiple concepts, a generalized viewpoint) and applications (how they are applied in a real life scenario).

- *Cooperative Learning in Undergraduate Mathematics Issues that Matter & Strategies that Work* by Elizabeth C. Rogers, Barbara E. Reynolds, Neil A. Davidson, Anthony D. Thomas, pages 56-71.
- *5 Practices for Orchestrating Productive Mathematics Discussion* by Margaret Schwan, Smith Mary, Kay Stein, pages 43-59.

- Article on "Aspects of Understanding: On Multiple Perspectives and Linear Representations and Connections Among Them" by Judit Moschkovich, Alan H. Schoenfeld and Abraham Arcavi.

Week 7:

Approaches to Cooperative Learning from various perspectives. How to ensure active thinking and participation by asking good questions and holding students accountable. Students will start to give presentations and classmates will give each other feedback. Asking good questions through productive questioning techniques that move discussion and ensure accountability. Students will focus their discussion on the teaching of the conceptual structure of Mathematics.

Required readings:

- *Cooperative Learning in Undergraduate Mathematics Issues that Matter & Strategies that Work* by Elizabeth C. Rogers, Barbara E. Reynolds, Neil A. Davidson, Anthony D. Thomas, pages 71-81.
- *5 Practices for Orchestrating Productive Mathematics Discussion* by Margaret Schwan, Smith Mary, Kay Stein, pages 61-73.
- Article on "Teaching the Conceptual Structure of Mathematics" by Lindsey E. Richland, James W. Stigler and Keith J. Holyoak.

Week 8:

Putting the Five Practices in the broader context of lesson planning. The details of developing thoughtful and thorough lesson plans (TTLP). The connection between TTLP and the five practices. Also a discussion of lesson planning beyond mastering the five practices. Students willcontinue to give presentations and get feedback from each other. The themes of the pedagogy class will continue to be discussed as the critiques of the presentations are based on the learning theory taught during the quarter.

Required readings:

- *5 Practices for Orchestrating Productive Mathematics Discussion* by Margaret Schwan, Smith Mary, Kay Stein, pages 75-84.
- "Providing Access to Meaningful Mathematics: Groupworthy Tasks," pages 35-45 from the book *Strength in Numbers: Collaborative Learning in Secondary Mathematics*.

Week 9:

Developing an understanding that students have pre-instructional conceptions when they enter the classroom. Effective instruction must take into account these conceptions as we work to further enhance students' knowledge base. To engage students with greater mathematical conceptions, we must have a clear understanding of their pre-existing knowledge and their mental models. Students will finish giving their short presentations.

Required reading:

- Various articles on students' ideas on specific topics in various disciplines will be posted on Canvas. The resource for these articles can be found in the Learning Assistant Alliance Resources (CU Boulder) under implementation for the Pedagogy course.

Week 10:

We will examine mental models and the specific role that they play in how students learn content that is presented in class. How to make use of the mental models as resources that can be used as the building blocks for obtaining further knowledge. The discussion will be focused on how to take advantage of the inherent strength that students brings as we teach them Precalculus, rather than undermining their self-confidence by marginalizing their precollege preparation. We will have a class activity where students will be asked to jot down specific characteristics of mental models such as thinking patterns developing through experience, patterns in one's mind, why ideas and procedures are better connected for certain mental models than others, etc.

Required reading:

- Article on "Implications of cognitive studies for teaching physics" by Redish, E. (1994), *American Journal of Physics*, (62) 9, Learning Assistant Alliance Resources (CU Boulder) under implementation for the Pedagogy course.

DRC Accommodations: UC Santa Cruz is committed to creating an academic environment that supports its diverse student body. If you are a student with a disability who requires accommodations to achieve equal access in this course, please submit your Accommodation Authorization Letter from the Disability Resource Center (DRC) to me privately during my office hours or by appointment, preferably within the first two weeks of the quarter. At that time, I would also like us to discuss ways we can ensure your full participation in the course. I encourage all students who may benefit from learning more about DRC services to contact DRC by phone at 831-459-2089 or by email at drc@ucsc.edu.

Academic Dishonesty

Plagiarism and Academic Dishonesty are extremely serious academic offences and in cases where investigation proves a significant departure from principles, penalties varying from partial or total loss of marks on the assignments or course to loss of scholarships, financial aid, to revocation of degree. Definitions, procedures, and penalties for dealing with plagiarism and academic dishonesty are set out in UCSC 2007-08 Academic Integrity on-line at: http://www.ucsc.edu/academics/academic_integrity/undergraduate_students/http://www.ucsc.edu/academics/academic_integrity/undergraduate_students/

List of Short Lecture Videos:

Video #1: Function and it's domain

Video #2: Piece-wise defined functions

Video #3: Graphing Techniques: Transformations

Video #4: Linear Functions

Video #5: Linear Function Modeling

Video #6: Solving Quadratic Equations using different methods

Video #7: Graphing of a Quadratic Function

Video #8: Quadratic Function Modeling

Video #9: Polynomial Function

Video #10: Rational Function

Video #11: Functions and its' Inverse

Video #12: Exponential Function

Video #13: Logarithmic Function

Video #14: Solving Exponential Equations

Video #15: Solve Logarithmic Equations

Video #16: Application with Log and Exponential Function

Video #17: Angles and their Measure

Video #18: Angular speed and Linear speed

Video #19: Defining basic six Trigonometric functions on the Unit Circle

Video #20: Finding Exact Values of the six trigonometric functions for the special angle $\theta = 45^0$ in all four quadrants of the unit circle

Video #21: Finding Exact Values of the six trigonometric functions for the special angle $\theta = 30^0$ in all four quadrants of the unit circle

Video #22: Finding Exact Values of the six trigonometric functions for the special angle $\theta = 60^0$ in all four quadrants of the unit circle

Video #23: Properties of Trigonometric Functions

Video #24: Graphs of functions of the form $y = A\sin(Bx - C) + \mathrm{D}$, where A, B, C and D are constants.

Video #25: Graphs of functions of the form $y = A\cos(Bx - C) + \mathrm{D}$, where A, B, C and D are constants.

Video #26: Basic graphs of $y = \tan x,\ y = \csc x,\ y = \sec x$

Video #27: The Inverse Sine Function

Video #28: The Inverse Tangent Function

Video #29: The Inverse Cosine Function

Video #30: Trigonometric Equations & Identities

Video #31: Sum and Difference Formulas

Video #32: Double Angle Formulas for the basic trig functions

Video #33: Right Triangle Trigonometry Application

Video #34: The Law of Sines and it's application

Video #35: The Law of Cosines and it's application

June 27, 2016

To Whom It May Concern:

I am writing this letter to provide details of my work with Nandini Bhattacharya on her grant-supported project to redesign Math 3.

Nandini contacted me in the winter of 2015 to ask for my input on her ideas and for help with a technology budget. As you know, her goal for the project was to increase student success through the provision of additional instruction. To that end, she designed a series of video lectures with accompanying quizzes. With the help of Leslie Kern, operations manager for the Faculty Instructional Technology Center, we built a small recording studio to record the lectures.

In the spring of 2015, Nandini and I began to produce the video lectures, in which she demonstrates concepts and procedures with writing and narration. During the first several weeks, Nandini devoted considerable time and effort to learning. In order to produce the lectures, she had to become familiar with new software (including the screen-recording program Camtasia) and hardware (including a Wacom tablet display). She also had to become comfortable with a new mode of instruction: the recorded lecture.

Additionally, Nandini used iClicker, another piece of new software, to quiz students on the material from the video lectures, and she chose to use the Canvas learning management system to deliver the videos. This was a great help to the group (of which I am a part) that managed the Canvas pilot at UCSC, as it allowed us to test the system with a high-enrollment course. Canvas was yet another piece of complex software that Nandini learned to use for this project.

In the spring of 2016, Nandini completed recording and producing the bulk of the video lectures. One of the next steps in the project is to develop short quizzes based on the lectures in Canvas to gauge students' understanding and to provide them with more opportunities for practice.

While this project is not yet complete, it is already successful. Student performance has improved, and student feedback on lecture videos and in-class quizzes with iClicker has been positive. It also provides a model for the redesign of other large lecture courses.

Sincerely,

Aaron Zachmeier
Instructional Designer

Spr Qtr 16

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Class Resources

- Math 3 TA and MSI Information.pdf
- MyMathLab Homework
- Resources for MyMathLab
- Syllabus Spring 2016-1.pdf
- MyMathLab Pearson eText
- Math 3 Sections.xlsx

Week 1

- Lecture Video 1 (Functions)
- Lecture Video 2 (Piecewise defined functi…
- Lecture Video 3 (Transformation of graphs)
- Lecture Video 4 (Linear models)
- Lecture Video 5 (Linear applications)
- Week 1 Notes.pdf

Week 2

- Lecture Video 6 (Solving Quadratic Equat…
- Lecture Video 7 (Graphing Quadratic Fun…

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Search for Assignment

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iClicker	5% of Total	+
MyMathLab	5% of Total	+
Quiz Score	10% of Total | 1 Rule	+
Midterm 1	20% of Total	+
Midterm 2	20% of Total	+
Final Exam	40% of Total	+